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PROPAGATION OF ELECTROMAGNETIC WAVES ALONG TWO PARALLEL SINGLE CONDUCTOR LINES

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[Figures referred to herein are appended.]

The problem of propagation of electromagnetic waves in a multiconductor system [1, 2, 3] is established, as we know, by the equations

$$-\frac{\partial}{\partial x}(V)=[R](J)+[L]\frac{\partial}{\partial t}(J). \quad -\frac{\partial}{\partial x}(J)=[G](V)+[C]\frac{\partial}{\partial t}(V)$$

e (V) and (J) - column matrices, whose elements $V_r(x,t)$ and $J_r(x,t)$ $(r=1,2,\dots,n)$ represent respectively the voltage in relation to the neutral conductor and the currents in the different conductors; [R], [L], [G] and [C] square matrices of the order n, of the resistance, inductance, conductivity of insulators and capacity respectively.

This problem was studied by K. Wagner [4] under the conditions that $n=2, [R]=0, [G]=0$ and the transfer of energy from one conductor to the other is unilateral; by L. Bryuley [2] under the condition that [R]=0, [G]=0 and n-optional, operation method was applied; by L. Pipes [3], who, under the condition that the line is symmetrized, resolved the problem by means of the definite integral from the Bessel function, applying the theory of matrices and Laplace's law; by V. I. Kovalenkov [5] who, with the following relation between the parameters of conductors

$$R_1/R_2=L_{11}/L_{22}=G_{22}/G_{11}=C_{22}/C_{11}=m^2/n^2$$

demonstrated that the general system of equation breaks down into ordinary telegraphic equations, when $n=2$.

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In this work the case when $n=2$ is examined. The solution of the problem is given in contour integrals. If A^2-4B is a complete square, integrals are expressed by the Lommel function.

1. Let us examine the ground and two conductors, to both ends of which are connected receiving - transmitting apparatus with feed at the nearest ends (Figure 1). Supposing that

$$V_1 = u_1,\ J_1 = u_2,\ V_2 = u_3,\ J_2 = u_4,\ L_{11} = L_1,\ L_{22} = L_2,\ L_{12} = M$$
$$G_{11} = G_1,\ G_{22} = G_2,\ G_{12} = -G_{12},\ C_{11} = C_1,\ C_{22} = C_2,\ C_{12} = -C_{12}$$

equations of propagation of electromagnetic waves will be

$$-\frac{\partial u_1}{\partial x} = R_1 u_2 + L_1 \frac{\partial u_2}{\partial t} + M \frac{\partial u_4}{\partial t}, \quad -\frac{\partial u_2}{\partial x} = G_1 u_1 + C_1 \frac{\partial u_1}{\partial t} - G_{12} u_3 - C_{12} \frac{\partial u_3}{\partial t}$$
$$-\frac{\partial u_3}{\partial x} = R_2 u_4 + L_2 \frac{\partial u_4}{\partial t} + M \frac{\partial u_2}{\partial t}, \quad -\frac{\partial u_4}{\partial x} = G_2 u_3 + C_2 \frac{\partial u_3}{\partial t} - G_{12} u_1 - C_{12} \frac{\partial u_1}{\partial t} \qquad (1.1)$$

These equations will be resolved in the interval $0<x<l$ with the time $t>0$ and under the initial conditions

$$u_1(x,0) = u_2(x,0) = 0, \qquad u_3(x,0) = u_4(x,0) = 0 \qquad (1.2)$$

Generally speaking, conditions limiting the line may be different, and, on the basis of the Kirchhof laws they are given by a system of linear differential equations.

In order to determine the type of equation (1.1) we shall establish the characteristic form [6] with the variables $y = \partial/\partial t,\ s = \partial/\partial x$:

$$C(y) = \left(s + \frac{y}{v_1}\right)\left(s - \frac{y}{v_1}\right)\left(s + \frac{y}{v_2}\right)\left(s - \frac{y}{v_2}\right) = 0$$

where

$$\frac{1}{v_i} = \sqrt{\frac{L_1C_1 + L_2C_2 - 2MC_{12}}{2} \pm \frac{1}{2}\sqrt{(L_1C_1 - L_2C_2)^2 + 4(L_1C_{12} - C_2M)(L_2C_{12} - C_1M)}} \qquad (1.3)$$

in which for $i = 1$ there will be a $+$, for $i = 2$, a $-$.

We shall further suppose that expressions (1.3) are always real. In that case, the four roots of the equation of the characteristic cone $C(y)$ are real and, therefore, the system of equations (1.1) is absolutely hyperbolic.

The problem examined is related to the composite problems of the second type: the initial conditions are similar, and limit conditions dissimilar (nonstationary problem [6]). In this case the solution is found by means of Laplace's law [7,8]. Multiplying equations (1.1) by e^{-pt}, supposing that $Re(p)>0$, let us integrate on t from 0 to ∞. Taking into account the initial conditions (1.2), we shall have for u_k an auxiliary system of linear equations with constant coefficients.

$$\frac{d\bar{u}_1}{dx} + z_1\bar{u}_2 + z_{12}\bar{u}_4 = 0, \qquad \frac{d\bar{u}_2}{dx} + w_1\bar{u}_1 - w_{12}\bar{u}_3 = 0$$
$$\frac{d\bar{u}_3}{dx} + z_{12}\bar{u}_2 + z_2\bar{u}_4 = 0, \qquad \frac{d\bar{u}_4}{dx} - w_{12}\bar{u}_1 + w_2\bar{u}_3 = 0 \qquad (1.4)$$

where

$$z_1 = R_1 + pL_1, \quad z_2 = R_2 + pL_2, \quad z_{12} = pM$$
$$w_1 = G_1 + pC_1, \quad w_2 = G_2 + pC_2, \quad w_{12} = G_{12} + pC_{12} \qquad \bar{u}_k(x,p) = \int_0^\infty u_k(x,t)e^{-pt}dt$$

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Applying the same transformation to the differential equations establishing the limit conditions we shall have

$$\bar{u}_1(0,p)=\bar{f}_1(p)-Z_{11}(p)\bar{u}_2(0,p), \quad \bar{u}_1(l,p)=Z_{12}(p)\bar{u}_2(l,p) \tag{1.5}$$

$$\bar{u}_3(0,p)=\bar{f}_2(p)-Z_{21}(p)\bar{u}_4(0,p), \quad \bar{u}_3(l,p)=Z_{22}(p)\bar{u}_4(l,p)$$

Here, $Z_{11}(p), Z_{12}(p), Z_{21}(p), Z_{22}(p)$ are the impedances of the receiving and transmitting apparatus and

$$\bar{f}_1(p)=\int_0^\infty f_1(t)e^{-pt}dt, \quad \bar{f}_2(p)=\int_0^\infty f_2(t)e^{-pt}dt$$

where $f_1(t)$ and $f_2(t)$ are functions expressing the law of variation of voltage at the generator's terminals.

By determining, from the limit conditions (1.5), the four constants entering into the solution of the system (1.4) we shall find the solution of $\bar{u}_k(x,p)$, then following the theorem of conversion for the Laplace integral, we shall also determine $u_k(x,t)$. Thus, we shall have the formal solution of the problem. In each specific case it is necessary to show that the formal solution exists and satisfies the equations and the initial and limit conditions.

2. Let us examine propagation of electromagnetic waves along two parallel, semi-infinite, homogeneous, single-wire lines, with initial current and voltage both equal to zero.

The nearest end of the first wire is subjected to a voltage equal to unity, while the nearest end of the second wire is grounded (Figure 2).

This problem leads to the solution of equations (1.1) in the semi-interval $0<x<\infty$ with $t>0$ under initial conditions (1.2) and the limit conditions being

$$u_1(0,t)=1, \quad u_3(0,t)=0 \tag{2.1}$$

Utilizing the method set forth in paragraph 1 the solution would appear under the form

$$u_k(x,p)=A_1\Delta_k(-r_1)e^{-r_1x}+A_2\Delta_k(r_1)e^{r_1x}+A_3\Delta_k(-r_2)e^{-r_2x}+A_4\Delta_k(r_2)e^{r_2x} \tag{2.2}$$

Here A_k (K = 1, 2, 3, 4) represent arbitrary constants, and

$$\Delta_1(r)=r^3+(z_{12}w_{12}-z_2w_2)r, \quad \Delta_2(r)=-w_1r^2+z_2(w_1w_2-w_{12}^2) \tag{2.3}$$

$$\Delta_3(r)=(z_{12}w_1-z_2w_{12})r, \quad \Delta_4(r)=w_{12}r^2-z_{12}(w_1w_2-w_{12}^2)$$

$$r_1=\sqrt{-\frac{A}{2}+\frac{1}{2}\sqrt{A^2-4B}}, \quad r_2=\sqrt{-\frac{A}{2}-\frac{1}{2}\sqrt{A^2-4B}} \tag{2.4}$$

represent adjunctions and roots with positive parts of the characteristic equation $p^4+Ap^2+B=0$, whereupon

$$A=2z_{12}w_{12}-z_1w_1-z_2w_2, \quad B=(z_1z_2-z_{12}^2)(w_1w_2-w_{12}^2) \tag{2.5}$$

$$A^2-4B=(2z_{12}w_{12}-z_1w_1-z_2w_2)^2-4(z_1z_2-z_{12}^2)(w_1w_2-w_{12}^2)=$$

$$=(z_1w_1-z_2w_2)^2+4(z_1w_{12}-z_{12}w_2)(z_2w_{12}-z_{12}w_2) \tag{2.6}$$

The final solution of the system (2.2) with $x\to\infty$ will be

$$\bar{u}_k(x,p)=A_1\Delta_k(-r_1)e^{-r_1x}+A_3\Delta_k(-r_2)e^{-r_2x} \quad (k=1,2,3,4) \tag{2.7}$$

By determining A_1 and A_3 from the limit conditions (2.1) we shall have

$$\bar{u}_1=\frac{1}{2p}(1+\mu)e^{-r_1x}+\frac{1}{2p}(1-\mu)e^{-r_2x} \quad \left(\mu=\frac{z_1w_1-z_2w_2}{\sqrt{A^2-4B}}\right)$$

$$\bar{u}_2=\frac{1}{p}\left[\frac{w_1}{2r_1}(1+\mu)+\frac{w_{12}}{r_1}\mu_1\right]e^{-r_1x}+\frac{1}{p}\left[\frac{w_1}{2r_2}(1-\mu)-\frac{w_{12}}{r_2}\mu_1\right]e^{-r_2x}$$

$$\bar{u}_3=-\frac{\mu_1}{p}e^{-r_1x}+\frac{\mu_1}{p}e^{-r_2x} \quad \left(\mu_1=\frac{z_2w_{12}-z_{12}w_1}{\sqrt{A^2-4B}}\right) \tag{2.8}$$

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$$\bar{u}_4 = -\frac{1}{p}\left[\frac{w_{12}}{2r_1}(1+\mu)+\frac{w_2}{r_1}\mu_1\right]e^{-r_1 x} - \frac{1}{p}\left[\frac{w_{12}}{2r_2}(1-\mu)-\frac{w_2}{r_2}\mu_1\right]e^{-r_2 x}$$

Finally, applying [illegible] theorem of transformation we shall have

$$u_1(x,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{1+\mu}{2}e^{pt-r_1 x}\frac{dp}{p} + \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{1-\mu}{2}e^{pt-r_2 x}\frac{dp}{p}$$

$$u_2(x,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\left[\frac{w_1}{2r_1}(1+\mu)+\frac{w_{12}}{r_1}\mu_1\right]e^{pt-r_1 x}\frac{dp}{p} +$$

$$+\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\left[\frac{w_2}{2r_2}(1-\mu)-\frac{w_{12}}{r_2}\mu_1\right]e^{pt-r_2 x}\frac{dp}{p}$$

$$u_3(x,t) = -\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\mu_1 e^{pt-r_1 x}\frac{dp}{p} + \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\mu_1 e^{pt-r_2 x}\frac{dp}{p} \qquad (2.9)$$

$$u_4(x,t) = -\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\left[\frac{w_{12}}{2r_1}(1+\mu)+\frac{w_2}{r_1}\mu_1\right]e^{pt-r_1 x}\frac{dp}{p} -$$

$$-\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\left[\frac{w_{12}}{2r_2}(1-\mu)-\frac{w_2}{r_2}\mu_1\right]e^{pt-r_2 x}\frac{dp}{p}$$

where a is such that all the characteristic points of $\bar{u}_k$ on the left side of the straight line representing $Re(p) = a$.

If the expression (2.6) is an absolute square, roots of equation (2.4) contain one radical and contour integrals (2.9) are expressed by means of functions of Lommel, starting from two imaginary independent variables [9].

We may point out that cases which were examined previously are particular cases of the correlations (2.6). Indeed, it is easy to ascertain that $A^2 - 4B$ is a complete square in the following cases: (1) single-conductor line [9, 10], $z_{12} = 0, w_{12} = 0$; (2) when for the symmetrized line [3], $z_1 = z_2, w_1 = w_2$; and (3) in the case examined by V. I. Kovelenkov [5] $z_1 w_1 = z_2 w_2$.

If $A^2 - 4B$ is not an absolute square, the problem for the time being is to be solved by means of numerical methods.

3. We will show that the formal solution obtained satisfies the problem's conditions. The functions u_k in the plane p have the following characteristics: a pole of the first order at the origin of the coordinates and eight radiation points determined by the equations $A^2 - 4B = 0$, $B = 0$. Supposing that coefficients of p^4 in these equations are different from zero, the radiation points will be located in the terminal part of the plane p, i.e., there is a circle of terminal radius R_o, with its center at the coordinates' origin, and containing all the characteristics of the function $\bar{u}_k$ (Figure 3).

Let us examine the contour L, representing the entire straight line $Re(p) = a$, parallel to the imaginary axis and located in the right semiplane and at a distance a from this axis.

Let us designate by $u_{k1}(L)$ and $u_{k2}(L)$, the integrals represented in the sum of the right member of (2.9), where in parentheses the contour of integration is indicated, and where the second index indicates to which item it is related. Utilizing the asymptotic representation of the roots r_1 and r_2 of the characteristic equation

$$r_1 = p/v_1 + r_{1p}, \qquad r_2 = p/v_2 + r_{2p}$$

where v_1 and v_2 are determined according to (1.3) and

$$\lim_{|p|\to\infty} r_{1p} = O(1), \qquad \lim_{|p|\to\infty} r_{2p} = O(1)$$

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we shall easily obtain the expressions of the exponents in the contour integrals (2.9):

$$pt - r_1 x = p\left(t - \frac{x}{v_1}\right) - r_{1p}x, \quad pt - r_2 x = p\left(t - \frac{x}{v_2}\right) + r_{2p}x$$

With the aid of these representations, it is easy to convince oneself that the multiples standing before exp p $(t - x/v_1)$ dp and exp p $(t - x/v_2)$ dp of subintegral expressions (2.9), beginning with a certain $R > R_0$, satisfy the conditions of the basic lemma of operational calculation [7, 8].

If $t - x/v_1 < 0$, the integration must be done by the contour ABCA. In the zone limited by this contour, function $\bar{u}_{K1}$ does not have any peculiarities; consequently, integral u_{K1} (ABCA) is equal to zero for any R. But u_{K1} (ABCA) = u_{K1} (ABC) + u_{K1} (CA) and according to the lemma lim u_{K1} (ABC) = 0 with $R \to \infty$ and, therefore, lim u_{K1} (CA) = 0 with $R \to \infty$, that is u_{K1} (L) = 0.

If $t - x/v_1 > 0$, the integration is done by contour ADCA. In the zone limited by this contour, functions $\bar{u}_{K1}$ have a special character. Therefore, integral u_{K1} (ADCA) is different from zero and remains the same for any $R > R_0$.

If we note that u_{K1} (ADCA) = u_{K1} (ADC) + u_{K1} (CA) and that according to the lemma lim u_{K1} (ADC) = 0 with $R \to \infty$, accordingly we have lim u_{K1} (CA) $\neq$ 0 with $R \to \infty$, that is u_{K1} (L) $\neq$ 0. Consequently, the integrals u_{K1} (L) have disruptions, corresponding to wave fronts spreading with velocity v_1, determined by the formula (1.3).

In making analogous analyses for u_{K2} (L), we find that in this case the propagation velocity of wave fronts v_2 is determined by (1.3).

Therefore, the solution (2.9) is twice disrupted. In addition, if $t - x/v_1 < 0$, $t - x/v_2 < 0$, then not one of the waves has reached the point under consideration. However, if $t - x/v_1 < 0$ and $t - x/v_2 > 0$ or vice versa, then the wave u_{K2} has reached the point, and wave u_{K1} has not, or vice versa. If, however, $t - x/v_1 > 0$ and $t - x/v_2 > 0$, then both waves have reached this point.

Let us prove that solution u_K (L) satisfies the equations (1.1). We shall first look at integrals u (L). If $t - x/v$ 0, we take integral u (ABCA); as the subintegral expressions of integral u (ABCA) are holomorphic functions, they can be differentiated according to parameters x and t. In substituting u_{K1} (ABCA) in equations (1.1), it is easy to convince oneself that they will be transformed into an identity for any R, and consequently, in making use of the lemma, we find that integrals u_{K1} (L) satisfy (1.1). In an analogous manner, we can prove that integrals u_{K1} (L) with $t - x/v_1 > 0$ and u_{K2} (L) with $t - x/v_2 \gtrless 0$ satisfy the equations (1.1).

Let us prove that functions u_1 (L) and u_3 (L) satisfy the limit conditions (2.1). Actually, with $x = 0$ we have

$$u_1(0,t) = \frac{1}{2\pi i}\int_L e^{pt}\frac{dp}{p} = 1, \quad u_3(0,t) = 0$$

Finally, let us show that solution u_K (L) satisfies the initial conditions (1.2). As a matter of fact, with $t = 0$ we have for u_1

$$u_1(x,0) = \frac{1}{2\pi i}\int_L \frac{1+\mu}{2} e^{-r_1 x}\frac{dp}{p} + \frac{1}{2\pi i}\int_L \frac{-\mu}{2} e^{-r_2 x}\frac{dp}{p}$$

As Re $(r_1) > 0$ and Re $(r_2) > 0$, we find, in carrying out an analysis analogous to those mentioned above, that $u_1(x, 0) = 0$.

In an analogous way, we can prove that the solution satisfies the other initial conditions.

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4. By way of example, let us examine a symmetrized line (3), that is, $z_1 = z_2 = z$ and $w_1 = w_2 = w$. In this case, we have $\mu = 0$, $\mu_1 = \frac{1}{2}$, and

$$r_1 = \frac{1}{v_1}\sqrt{(p+2\alpha_1)(p+2\beta_1)}\,, \qquad r_2 = \frac{1}{v_2}\sqrt{(p+2\alpha_2)(p+2\beta_2)} \tag{4.1}$$

where

$$\frac{1}{v_1} = \sqrt{(L-M)(C+C_{12})}, \quad \alpha_1 = \frac{R}{2(L-M)}, \quad \beta_1 = \frac{G+G_{12}}{2(C+C_{12})}$$

$$\frac{1}{v_2} = \sqrt{(L+M)(C-C_{12})}, \quad \alpha_2 = \frac{R}{2(L+M)}, \quad \beta_2 = \frac{G-G_{12}}{2(C-C_{12})} \tag{4.2}$$

and the integrals (2.0) will appear as follows

$$u_1 = \frac{1}{2}\left\{\frac{1}{2\pi i}\int_{\gamma} e^{pt-r_1 x}\frac{dp}{p} + \frac{1}{2\pi i}\int_{\gamma} e^{pt-r_2 x}\frac{dp}{p}\right\}$$

$$u_2 = \frac{1}{2}\left\{\sqrt{\frac{C+C_{12}}{L-M}}\,\frac{1}{2\pi i}\int_{\gamma}\sqrt{\frac{p+2\beta_1}{p+2\alpha_1}}\,e^{pt-r_1 x}\frac{dp}{p} + \right.$$

$$\left. + \sqrt{\frac{C-C_{12}}{L+M}}\,\frac{1}{2\pi i}\int_{\gamma}\sqrt{\frac{p+2\beta_2}{p+2\alpha_2}}\,e^{pt-r_2 x}\frac{dp}{p}\right\} \tag{4.3}$$

$$u_3 = \frac{1}{2}\left\{-\frac{1}{2\pi i}\int_{\gamma} e^{pt-r_1 x}\frac{dp}{p} + \frac{1}{2\pi i}\int_{\gamma} e^{pt-r_2 x}\frac{dp}{p}\right\}$$

$$u_4 = \frac{1}{2}\left\{-\sqrt{\frac{C+C_{12}}{L-M}}\,\frac{1}{2\pi i}\int_{\gamma}\sqrt{\frac{p+2\beta_1}{p+2\alpha_1}}\,e^{pt-r_1 x}\frac{dp}{p} + \right.$$

$$\left. + \sqrt{\frac{C-C_{12}}{L+M}}\,\frac{1}{2\pi i}\int_{\gamma}\sqrt{\frac{p+2\beta_2}{p+2\alpha_2}}\,e^{pt-r_2 x}\frac{dp}{p}\right\}$$

These integrals were previously calculated [9, 10] and appear as follows:

$$u_1 = \frac{1}{2}(u_{11}+u_{12}), \qquad u_2 = \frac{1}{2}(u_{21}+u_{22})$$

$$u_3 = \frac{1}{2}(u_{11}+u_{12}), \qquad u_4 = \frac{1}{2}(-u_{21}+u_{22}) \tag{4.4}$$

Here,

$$u_{11} = e^{-\rho_1 t}\left[I_0(\zeta_1) + r_1(\xi_1,\zeta_1) + r_2(\xi_1,\zeta_1) + r_1(\eta_1,\zeta_1) + r_2(\eta_1,\zeta_1)\right]H\left(t-\frac{x}{v_1}\right)$$

$$u_{12} = e^{-\rho_2 t}\left[I_0(\zeta_2) + r_1(\xi_2,\zeta_2) + r_2(\xi_2,\zeta_2) + r_1(\eta_2,\zeta_2) + r_2(\eta_2,\zeta_2)\right]H\left(t-\frac{x}{v_2}\right)$$

$$u_{21} = \sqrt{\frac{G+G_{12}}{R}}\,e^{-\rho_1 t}\left[\sqrt{\frac{\alpha_1}{\beta_1}}\,I_0(\zeta_1) + r_1(\xi_1,\zeta_1) + r_2(\xi_1,\zeta_1) - \right. \tag{4.5}$$

$$\left. - r_1(\eta_1,\zeta_1) - r_2(\eta_1,\zeta_1)\right]H\left(t-\frac{x}{v_1}\right)$$

$$u_{22} = \sqrt{\frac{G-G_{12}}{R}}\,e^{-\rho_2 t}\left[\sqrt{\frac{\alpha_2}{\beta_2}}\,I_0(\zeta_2) + r_1(\xi_2,\zeta_2) + r_2(\xi_2,\zeta_2) - \right.$$

$$\left. - r_1(\eta_2,\zeta_2) - r_2(\eta_2,\zeta_2)\right]H\left(t-\frac{x}{v_2}\right)$$

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where

$$\rho_1 = \alpha_1 + \beta_1, \quad \sigma_1 = \alpha_1 - \beta_1, \quad m_1 = \sqrt{\alpha_1} + \sqrt{\beta_1}, \quad n_1 = \sqrt{\alpha_1} - \sqrt{\beta_1}$$

$$\rho_2 = \alpha_2 + \beta_2, \quad \sigma_2 = \alpha_2 - \beta_2, \quad m_2 = \sqrt{\alpha_2} + \sqrt{\beta_2}, \quad n_2 = \sqrt{\alpha_2} - \sqrt{\beta_2}$$

$$\xi_1 = m_1^2\left(t - \frac{x}{v_1}\right), \quad \eta_1 = n_1^2\left(t - \frac{x}{v_1}\right), \quad \zeta_1 = \sigma_1\sqrt{t^2 - \left(\frac{x}{v_1}\right)^2}$$

$$\xi_2 = m_2^2\left(t - \frac{x}{v_2}\right), \quad \eta_2 = n_2^2\left(t - \frac{x}{v_2}\right), \quad \zeta_2 = \sigma_2\sqrt{t_2 - \left(\frac{x}{v_2}\right)^2}$$

and $H(y) = 0$ with $y < 0$, $H(y) = 1$ with $y > 0$.

In examining the formulas (4.4) we note that the inducing wave of voltage u_1 consists of the half-sum of a fast and slow wave, and the induced wave of voltage u_3 consists of the half-difference of these same waves. With any interval of time t, the fast and slow waves of the voltages will separate at a distance $l = |v_1 - v_2|\,t$, and all along this distance the amplitudes of the inducing and induced fast wave remain equal as to their absolute value.

Let us show now that with M, G_{12}, C_{12}, approaching zero, there remains only the inducing wave of voltages whereas the induced wave will disappear. Actually, with this assumption according to formulas (4.1), (4.2), and (4.5), we have $\sigma_1 \to \alpha_2$, $\beta_1 \to \beta_2$, $v_1 \to v_2$, $r_1 \to r_2$, $u_1 \to u_{12}$ and out of (4.4) there results $u_1 \to u_{11}$, $u_3 \to 0$, which is what we had to prove.

All the cited conditions remain in force for waves of current.

For purposes of illustration, we show in Table 1 and Figure 4 the results of calculation for two single-conductor copper four-millimeter lines with the constants [11, 12], as follows:

Table 1

x km	u_1	$t = 3.37$ $10^3 u_2$	μsec u_3	$10^3 u_4$
0	1.000	1.143	0.000	-0.217
250	0.654	0.821	0.068	-0.103
500	0.420	0.586	0.093	-0.034
750	0.257	0.404	0.103	0.027
981-0	0.157	0.207	0.100	0.143
981+0	0.129	0.175	0.129	0.175
1000-0	0.125	0.172	0.125	0.172
1000+0	0.000	0.000	0.000	0.000

$R = 1.49\ \frac{\Omega}{km}$

$L = 17.1 \times 10^{-4}\ \frac{H}{km}$

$G = 1.5 \times 10^{-5}\ \frac{1}{\Omega\,km}$

$C = 8.36 \times 10^{-9}\ \frac{F}{km}$

$M = 7.36 \times 10^{-4}\ \frac{H}{km}$

$G_{12} = 0.5 \times 10^{-6}\ \frac{1}{\Omega\,km}$

$C_{12} = 3.72 \times 10^{-9}\ \frac{F}{km}$

In using the methods described in [13], we shall obtain a solution of the problem for lines with receiving transmitting facilities with any kind of generators at the near terminals.

In conclusion, I wish to express my thanks to V. I. Kovalenkov for presenting the problem, and to V. N. Kuznetsov and N. N. Luzin for valuable advice in completing this work.

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[Appended figures follow.]

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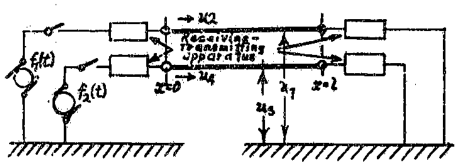

Figure 1.

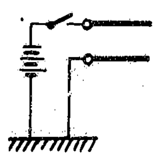

Figure 2.

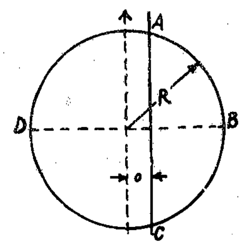

Figure 3.

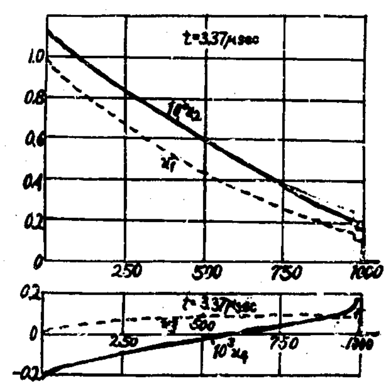

Figure 4.

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